

$$H = \begin{bmatrix} I & I & I & \dots & I \\ I & \sigma^1 & \sigma^2 & \dots & \sigma^{p-1} \\ I & \sigma^2 & \sigma^4 & \dots & \sigma^{2(p-1)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ I & \sigma^{r-1} & \sigma^{2(r-1)} & \dots & \sigma^{(r-1)(p-1)} \end{bmatrix}$$

FIG.1

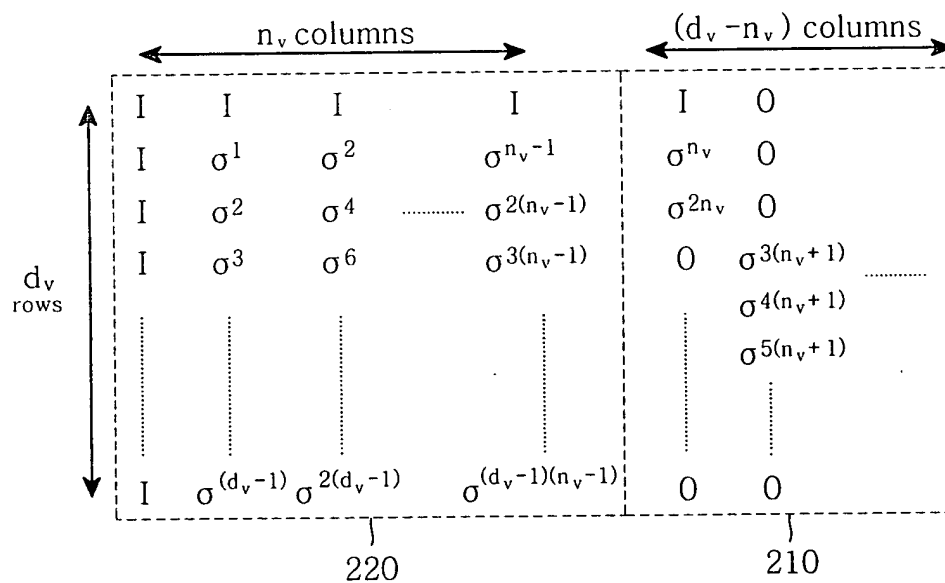


FIG.2

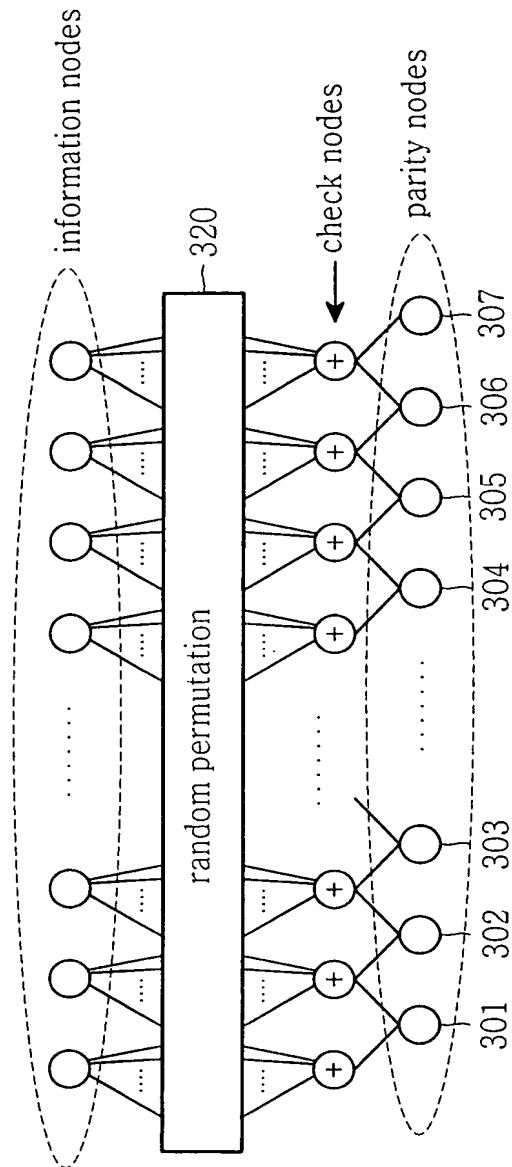


FIG.3

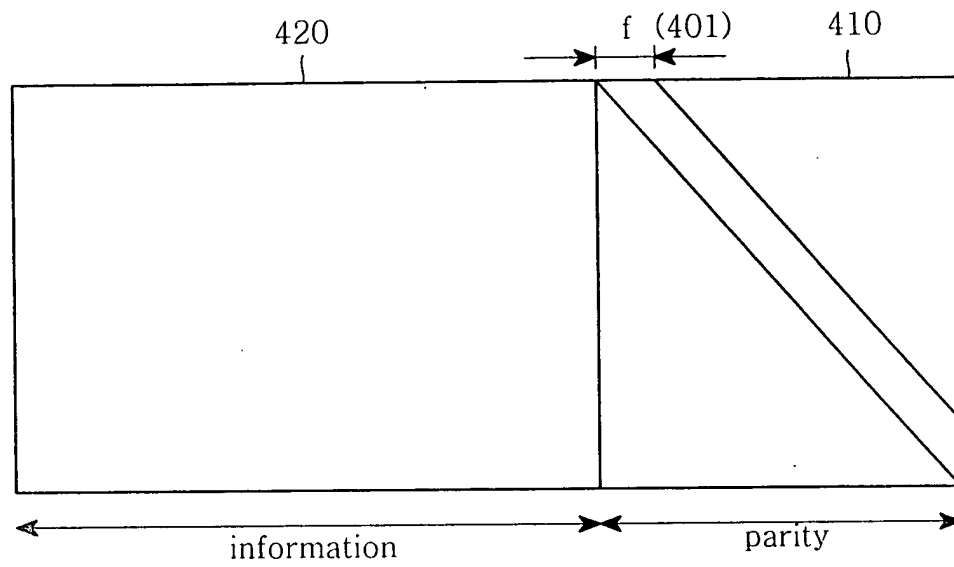


FIG.4

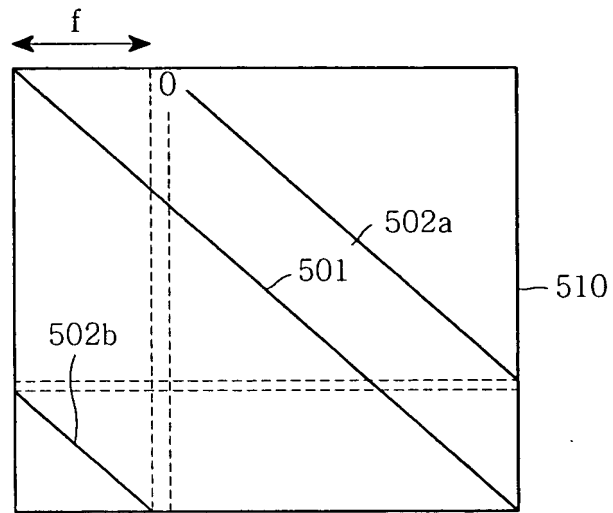


FIG. 5

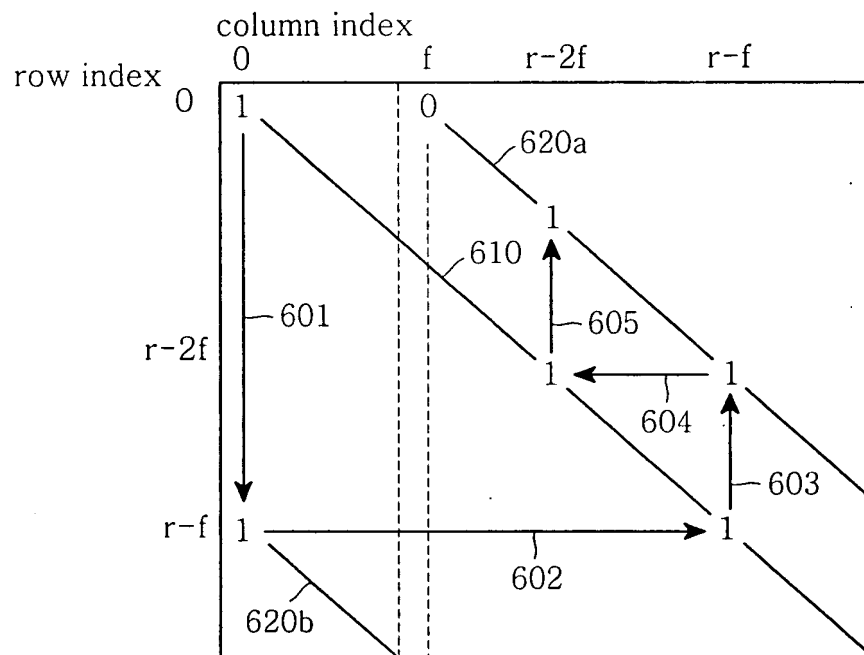


FIG. 6

0	0	0	0		0		0		
0	1	2	3		6			11	
0	2	4	6		12			22	
0	3	6	9		18			33	
0	4	8	12			28			48
0	5	10		20		35			60
0	6	12		24		42			72
0	7	14		28			56		2
0	8	16		32			64		15
0	9	18		36			72		28
0	10	20			50			1	51
0	11	22			55			10	65
0	12	24			60			19	79
0	13	26			65				41
0	14	28			70				51

Example of H_d with irregular distribution of $d_v = 15$ ($p=89$)

FIG.7

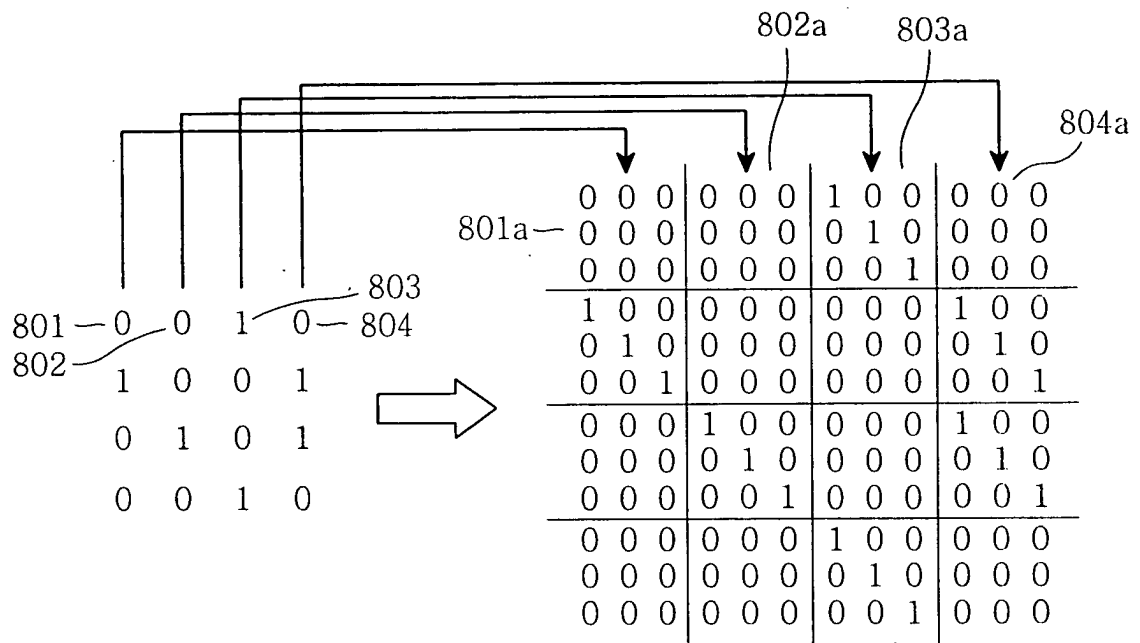


FIG.8

fsub-matrix columns(px f columns)

$$H_P = \begin{bmatrix} \sigma^{j_0} & & \sigma^{j_1} & & \\ & \sigma^{j_2} & & \sigma^{j_3} & \\ & & \sigma^{j_{2(r-f-1)}} & & \sigma^{j_{2(r-f-1)+1}} \\ \sigma^{j_{2(r-f)+1}} & & & \sigma^{j_{2(r-f)}} & \\ & & \sigma^{j_{2(r-1)+1}} & & \sigma^{j_{2(r-1)}} \end{bmatrix}$$

FIG.9

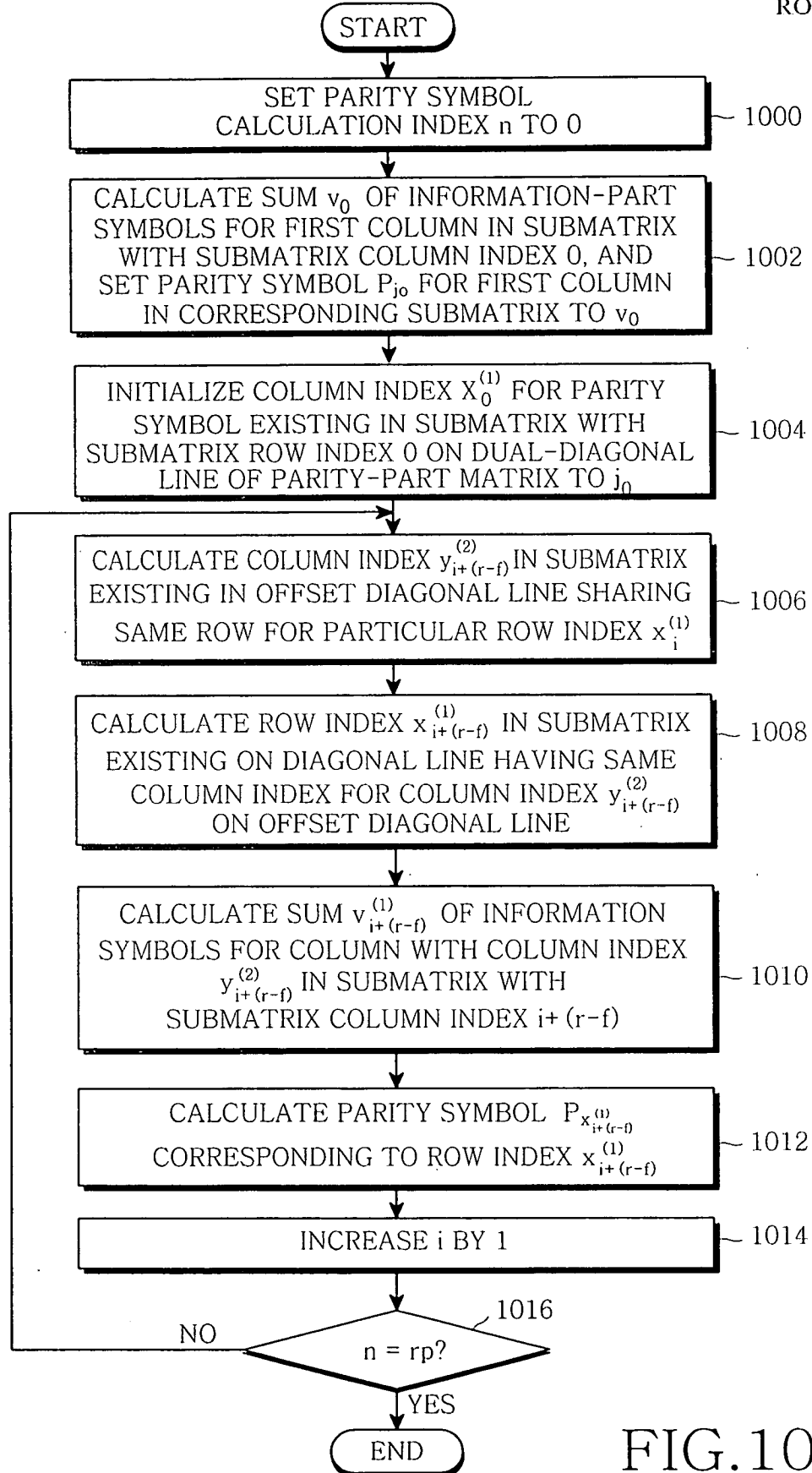
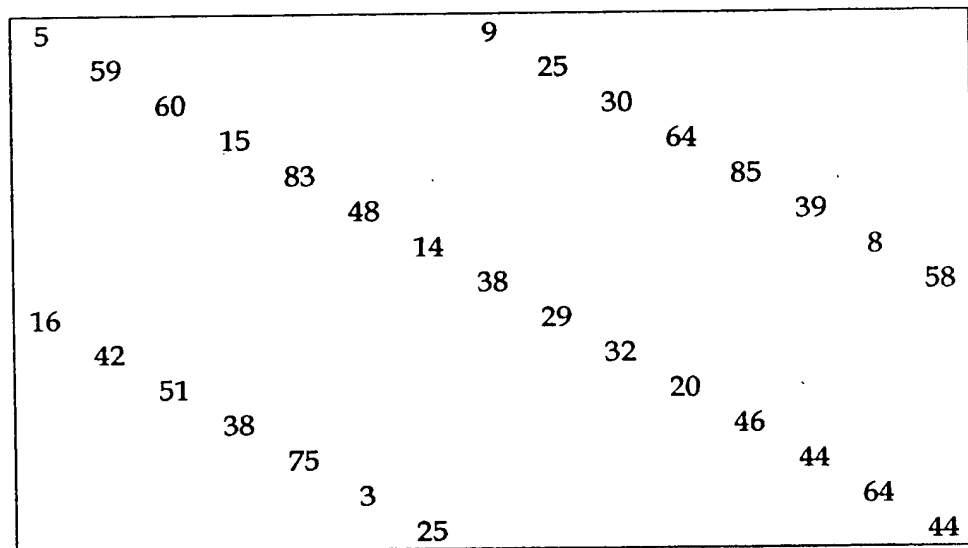


FIG. 10



Example of H_p by lifting the generalized dual-diagonal matrix ($r=15$, $f=7$, $p=89$)

FIG.11

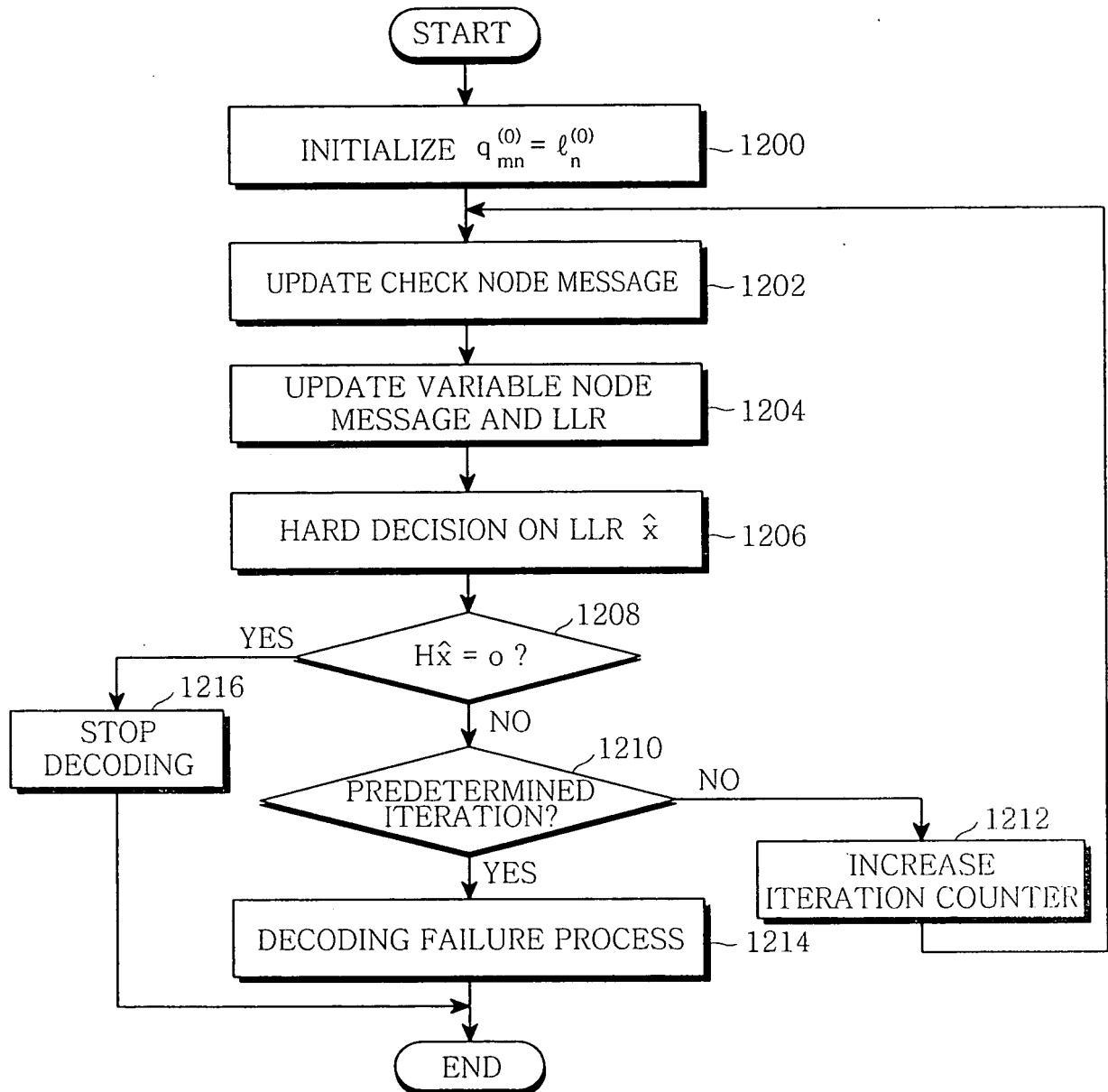


FIG.12

- H_d

0	0	0	0		0		0		
0	1	2	3		6			11	
0	2	4	6		12			22	
0	3	6	9		18			4	
0	4	8	12			28			19
0	5	10		20		6			2
0	6	12		24		13			14
0	7	14		28			27		4
0	8	16		3		6			17
0	9	18		7		14			1
0	10	20			21		3		24
0	11	22			26		12		9
0	12	24			2		21		23
0	13	26			7			14	
0	14	28			12			24	

FIG.13A

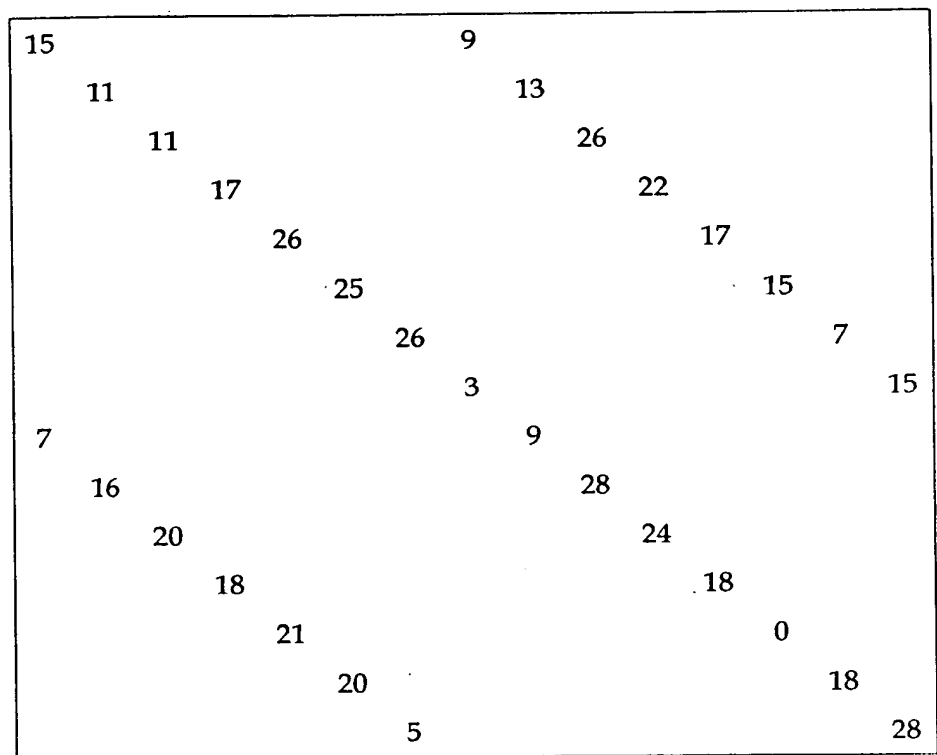
$-H_p$ 

FIG.13B

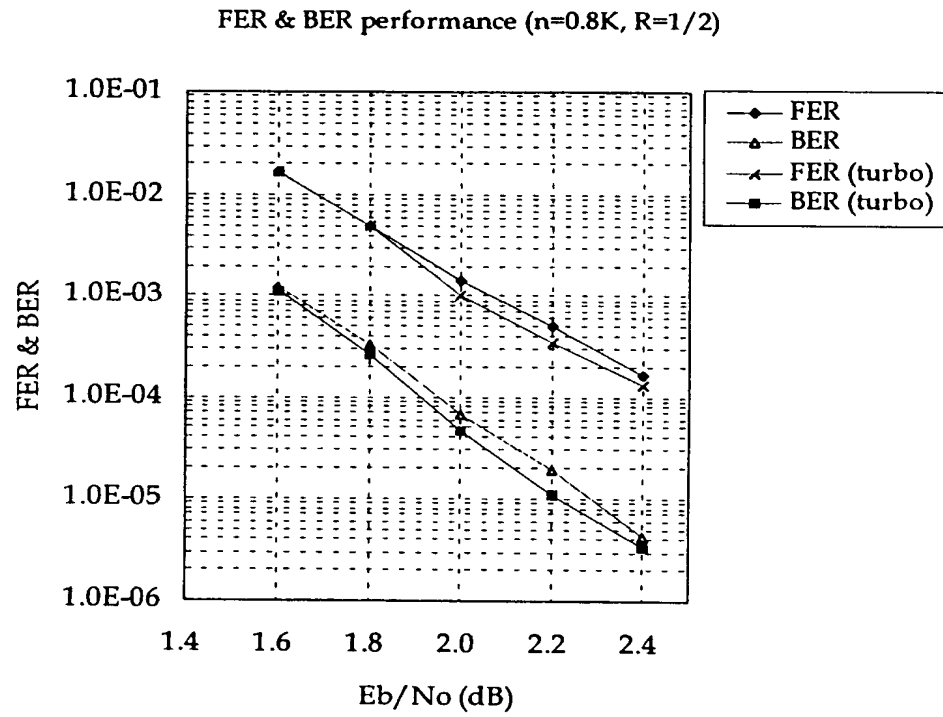


FIG.13C

- H_d

0	0	0	0		0		0		
0	1	2	3		6			11	
0	2	4	6		12			22	
0	3	6	9		18			33	
0	4	8	12			28			48
0	5	10		20		35			7
0	6	12		24		42			19
0	7	14		28			3		38
0	8	16		32			11		51
0	9	18		36			19		11
0	10	20			50			37	34
0	11	22			2			46	48
0	12	24			7			2	9
0	13	26			12			24	
0	14	28			17			34	

FIG.14A

The scatter plot displays 48 data points, each labeled with a number. The numbers range from 1 to 49, with some numbers appearing multiple times. The points are distributed across the plot area, with a higher density in the upper right quadrant. The following table lists the numbers of the points in the plot:

Point Number
1
12
13
14
15
20
25
28
31
33
35
37
38
40
41
44
46
47
49

FIG.14B

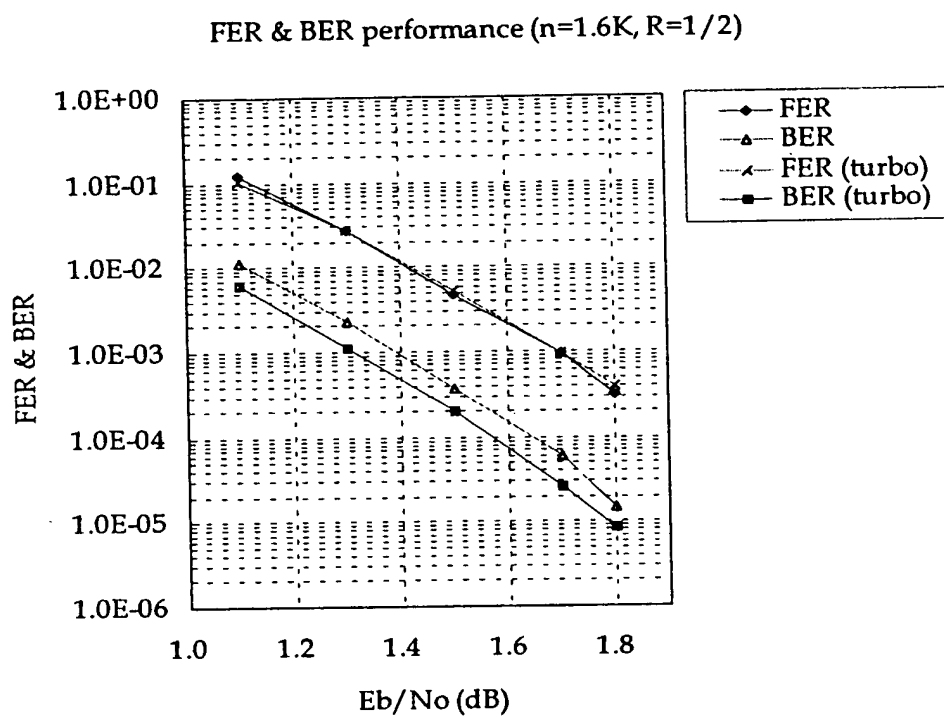


FIG.14C

- H_d

0	0	0	0		0		0		
0	1	2	3		6			11	
0	2	4	6		12			22	
0	3	6	9		18			33	
0	4	8	12			28			48
0	5	10		20		35			60
0	6	12		24		42			72
0	7	14		28			56		91
0	8	16		32			64		1
0	9	18		36			72		14
0	10	20			50			90	37
0	11	22			55			99	51
0	12	24			60			5	65
0	13	26			65			27	
0	14	28			70			37	

FIG.15A

- H_p

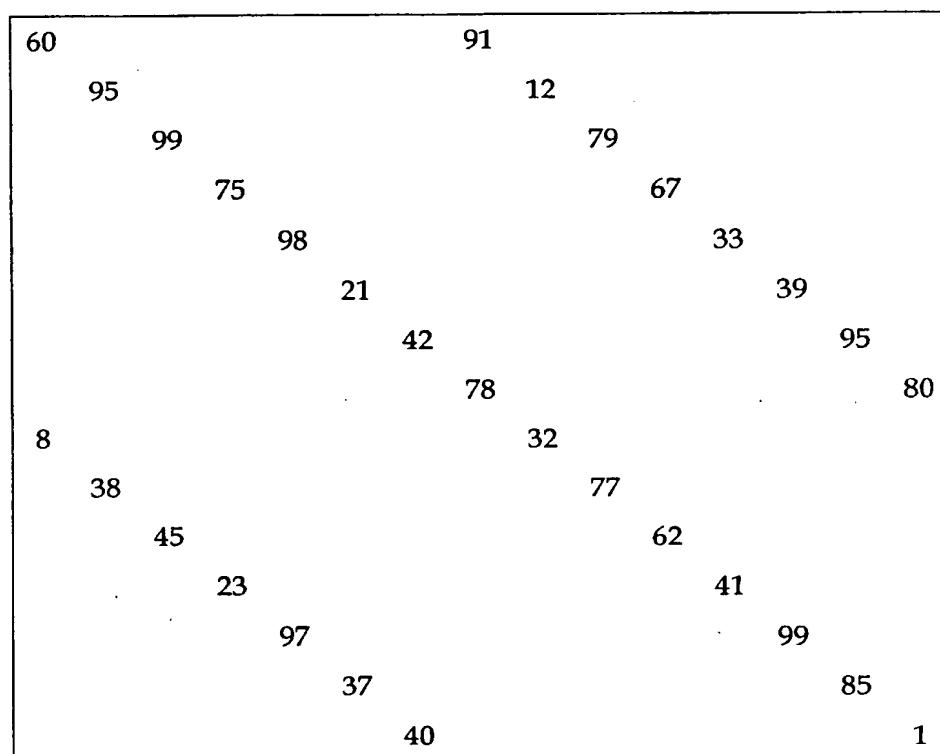


FIG.15B

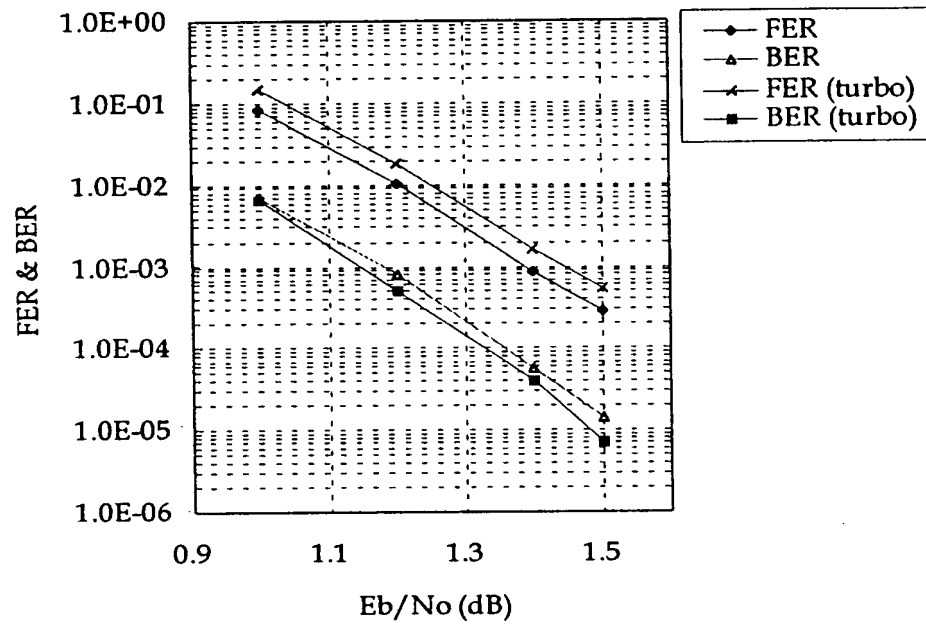
FER & BER performance ($n=3.1K$, $R=1/2$)

FIG.15C

- H_a

0	0	0	0	0	0	0
0	1	2	3	6	11	
0	2	4	6	12	22	
0	3	6	9	18	33	
0	4	8	12	28	48	
0	5	10	20	35	60	
0	6	12	24	42	72	
0	7	14	28	56	91	
0	8	16	32	64	104	
0	9	18	36	72	117	
0	10	20	50	90	140	
0	11	22	55	99	154	
0	12	24	60	108	168	
0	13	26	65	130		
0	14	28	70	140		

FIG.16A

0	0	0	0		0		0		
0	1	2	3		6			11	
0	2	4	6		12			22	
0	3	6	9		18			33	
0	4	8	12			28			48
0	5	10		20		35			60
0	6	12		24		42			72
0	7	14		28			56		2
0	8	16		32			64		15
0	9	18		36			72		28
0	10	20			50			1	51
0	11	22			55			10	65
0	12	24			60			19	79
0	13	26			65			41	
0	14	28			70			51	

Example of H_d with irregular distribution of $d_v = 15$ ($p=89$)

FIG.16B

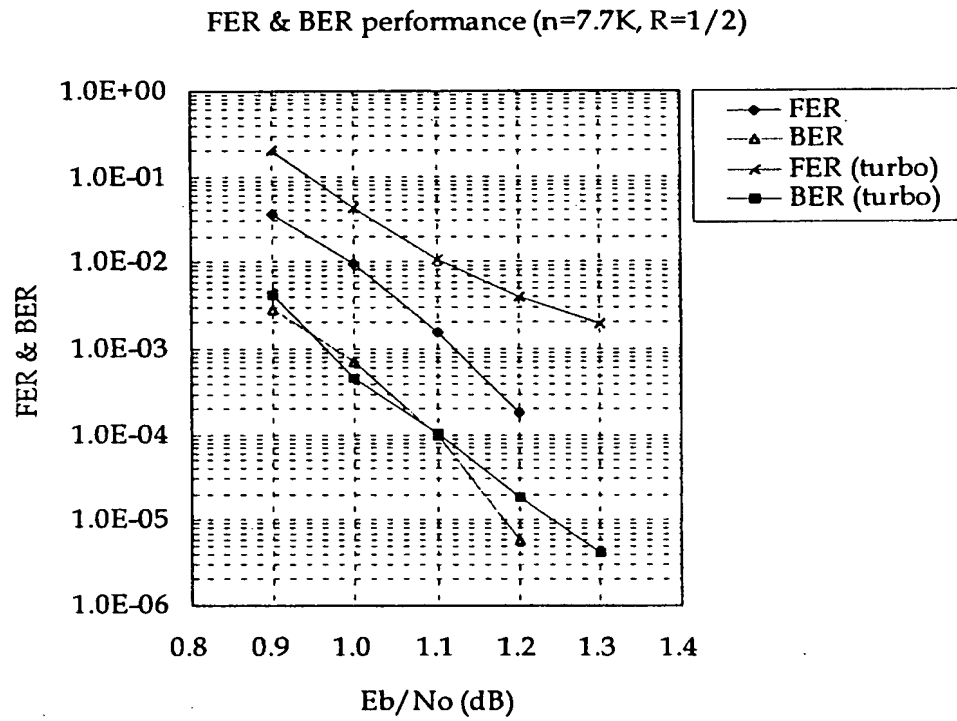


FIG.16C

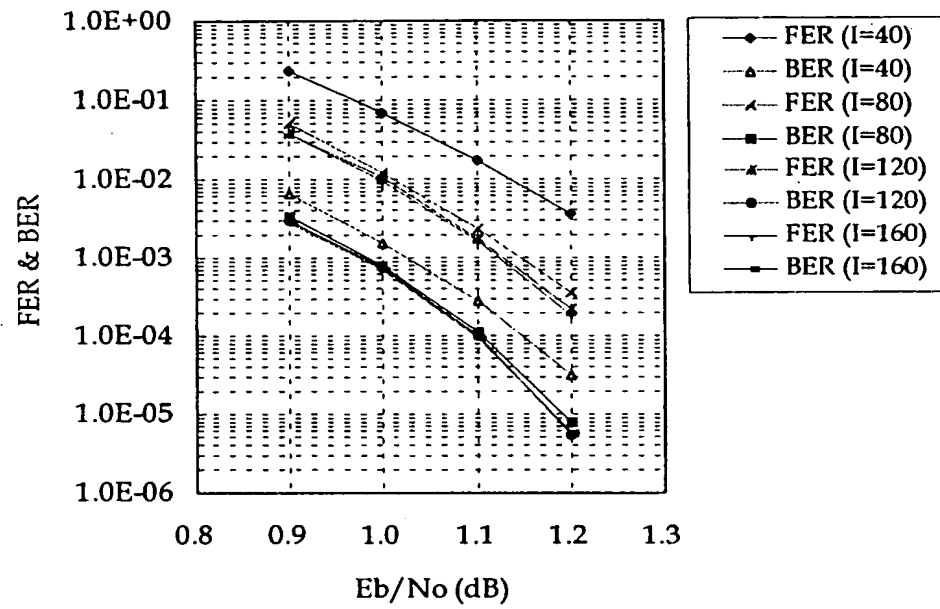
FER & BER performance ($n=7.7K$, $R=1/2$)

FIG.16D